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AN INSPECTION SYSTEM TO IDENTIFY FATIGUE DAMAGE ON STEEL-BRIDGE STRUCTURES

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Abstract. *Crack formation and growth in steel bridge structural elements may be due to loading oscillations. The welded elements are liable to internal discontinuities along welded joints and sensible to stress variations. The evaluation of the remaining life of a bridge is needed to make cost-effective decisions regarding inspection, repair, rehabilitation, and replacement. A steel beam model has been proposed to simulate crack openings due to cyclic loads. Two possible alternatives have been considered to model crack propagation, which the initial phase is based on the linear fracture mechanics. Then, the model is extended to take into account the elastoplastic fracture mechanic concepts. The natural frequency changes are directly related to moment of inertia variation and consequently to a reduction in the flexural stiffness of a steel beam. Thus, it is possible to adopt a nondestructive technique during steel bridge inspection to quantify the structure eigenvalue variation that will be used to localize the grown fracture. A damage detection algorithm is developed for the proposed model and the numerical results are compared with the solutions achieved by using another well know computer code.*

Keywords: *Fatigue evaluation, Steel bridges, Fracture mechanics*

1. INTRODUCTION

Fractures due fatigue develop in steel bridges as result of the repeated loading to which they are submitted. A number of failures occurring in steel structures as bridges, offshore platforms, ships, cranes, vehicles, aircraft and general equipments subjected to repeated loading result from the propagation of fatigue cracks. The occurrence of cracks is more frequently in welded bridge structures because of flaws that escaped detection and the variety of details. It is also known that the primary factors contributing to cracking in steel bridges are: type of detail, age of the bridge, frequency of the truck traffic, magnitude of stress range, quality of the fabricated detail and material fracture toughness. When a crack is found by inspection, it is important to predict further growth in order to decide the convenient time to repair the structure (Brinckerhoff, 1993). In order to accomplish this study, a system inspection model is developed. This model, based on finite element analysis and linear fracture mechanic concepts, is used during a nondestructive evaluation. In order to treat the crack identification problem, one should consider the fact that a crack formed in an element introduces a local stiffness reduction, which modify the dynamic behavior of the structure. In this paper, the problem of crack detection is formulated with reference to a simply supported beam and it is also assumed that the crack occurs in the opening mode (first mode of fracture). Linear elastic fracture mechanic is here employed to describe the initiation and the development of the fatigue crack in the steel beam. Furthermore, the results obtained in studies of various parameters that have an influence on the damage detection are given.

2. MATHEMATICAL MODEL AND FINITE ELEMENT ANALYSIS APPROACH

Let us consider the case of a beam with a discrete single crack (Mode I) at the centre. The beam, characterised by a constant cross section A and moment of inertia I_o, is loaded by moments M and axial force N applied at its ends. Let the displacement and rotation due to existence of the crack be δ and θ (see Fig. 1).

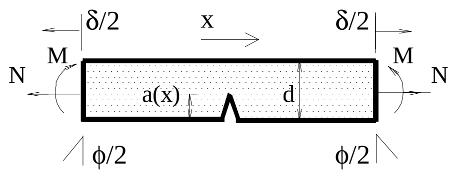


Figure1 – Single- edge cracked beam under combined axial force and bending.

It is well known that, The fatigue crack growth rate depends on the magnitude of the stress range $\Delta\sigma$, the crack length a and the material properties. Both, the stress range and the crack length, are used to compute stress intensity factor range ΔK_I. Then, the fatigue crack growth rate, da/dN, is related to ΔK_I by the empirical relationship proposed by Paris-Erdogan (Paris & Erdogan, 1960):

$$\frac{da}{dN} = C(\Delta K_I)^m \tag{1}$$

where N is the number of cycles and C, m are constants related with material properties and environments. Once a fatigue crack has initiated in a structure, the remaining life can be predicted by rearranging and integrating Eq.1 between the limits of initial and final crack lengths a_i and a_f (point where $K_{max} = K_{Ic}$):

In some complex situations due to geometry, such as I – sections, or to non-uniform growth of plane cracks one can better express the above law in term of energy release rate G instead of stress intensity factor K_I. Recalling that, by definition, the crack-extension force G is given by the change of the strain energy with respect to crack extension, one can also say:

$$G = \frac{\partial U^*}{\partial A} \tag{2}$$

where U* is the complementary energy of the structure and A stands for the crack area. According the Saint-Venant's principle, the stress field is affected only in the vicinity of the crack, ξ(a) (Fig. 2). Thus, one can assume that the value elastic modulus × inertia product, EI, at any beam cross section is given by:

$$EI(x,a) = \begin{cases} EI_o[1-\phi(x,a)] & for \quad 0 \le x \le \xi(a) \\ EI_o & for \quad x > \xi(a) \end{cases} \tag{3}$$

where ϕ(x,a) is a function depending on the cross section shape, such that $0 \le \phi(x,a) \le 1$.

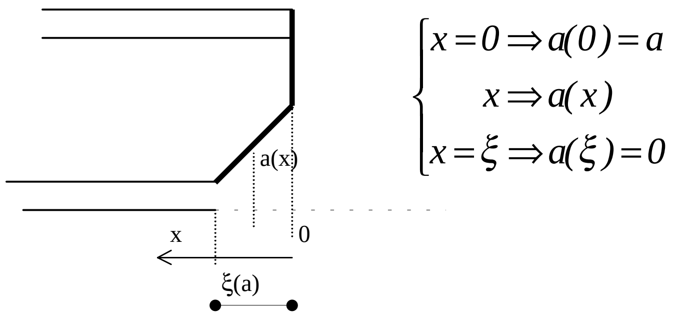


Figure 2 – Crack propagation.

In this case, the bending moments distribution is given by:

$$M(x) = M_i + \left(\frac{M_e - M_i}{l}\right)x + \left(\frac{N.a}{2l}\right)x - \frac{N}{2}\left(a - a(x)\right) \tag{4}$$

with $M_e = \kappa.M_i$ (κ = constant) and l is the length. The complementary energy U* of the beam between the cracking section and one of its side can be evaluated as:

$$U^* = U_o^* + \frac{1}{2}\int_o^{\xi(a)} \frac{M_{(x)}^2}{EI_o}\eta_M(x,a)dx + \frac{N^2}{2EA_o}\int_o^{\xi(a)}\eta_N(x,a)dx \tag{5}$$

with U_o^* representing the complementary energy and A_o the cross section in the undamaged situation, and

$$\eta_M(x,a) = \frac{\phi(x,a)}{1-\phi(x,a)} \tag{6}$$

$$\eta_N(x,a) = \frac{A_o - A}{A} \quad (7)$$

$$\begin{cases} A = A_o[1-\phi_N(x,a)] & ,0 \le x \le \xi(a) \\ A = A_o & , \quad x > \xi(a) \end{cases} \quad (8)$$

where $\phi_N(x,a)$ depends on the cross section shape, $0 \le \phi_N(x,a) \le 1$.

2.1 Global stiffness matrix evaluation

It can be easily seen that the third term of Eq. 5 disappear in the case of pure bending (κ=1). Thus, the complementary energy U* of the half beam can be computed as:

$$U^* = U_o^* + \frac{M_i^2}{2EI_o}[\Theta_1 + 2l^{-1}(\kappa-1)\Theta_2 + l^{-2}(\kappa-1)^2\Theta_3] \quad (9)$$

where U_o* is the complementary energy in the uncracked situation, M_e= κ. M_i (κ = constant) and:

$$\Theta_i = \int_0^{\xi(a)} x^{i-1}\eta(x,a)dx \quad (10)$$

with $\eta(x,a) = \phi(x,a)[1-\phi(x,a)]^{-1}$.

The second term of Eq. 9 can be considered as the complementary energy of a spring with rotational stiffness related with the degrading region:

$$k = \frac{EI_o}{\Gamma(x,a)} \quad (11)$$

where: $\Gamma(x,a) = [\Theta_1 + 2l^{-1}(\kappa-1)\Theta_2 + l^{-2}(\kappa-1)^2\Theta_3]$

The extension of Eq. (9) to any complex structure is straightforward, as a directly addiction of the complementary energy referring to each segment of the beam. The global stiffness matrix can be obtained by adding the neighbour elements whose linear system of algebraic equations are given by:

$$\begin{bmatrix} [K^{(1)}]_{5x5} & \{k^{(1)}\}_{5x1} \\ \{k^{(1)}\}^T_{1x5} & k^{(1)} \end{bmatrix} \cdot \begin{Bmatrix} u^{(1)} \\ \phi^{(1)} \end{Bmatrix} = \begin{Bmatrix} P^{(1)} \\ k*(\phi^{(2)} - \phi^{(1)}) \end{Bmatrix} \quad (12a)$$

$$\begin{bmatrix} k^{(2)} & \{k^{(2)}\}^T_{1x5} \\ \{k^{(2)}\}_{1x5} & [K^{(2)}]_{5x5} \end{bmatrix} \cdot \begin{Bmatrix} \phi^{(2)} \\ u^{(2)} \end{Bmatrix} = \begin{Bmatrix} k*(\phi^{(1)} - \phi^{(2)}) \\ P^{(2)} \end{Bmatrix} \quad (12b)$$

The coefficients given in equation (12) ($[K^{(i)}]$, $\{k^{(i)}\}$, $k^{(i)}$, i =1,2) are values derived for standard beam elements. After jointing the elements sides the remaining degrees of freedom of the cracked beam are (Fig. 3): $W = [w_1,w_2,w_3,w_4,w_5,w_6,w_7,w_8]^T$.

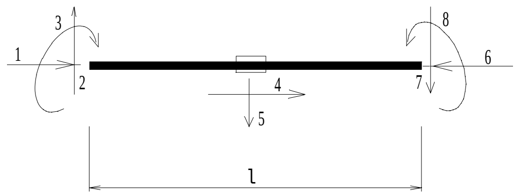


Figure 3 – Cracked element degrees of freedom.

2.2 Stress Intensity factor for the cracked beam

To evaluate the stress intensity factor K_I it is suggested an analogy with the opening procedure of a *zipper* (Deus, 1997) as shown in Fig. 4.

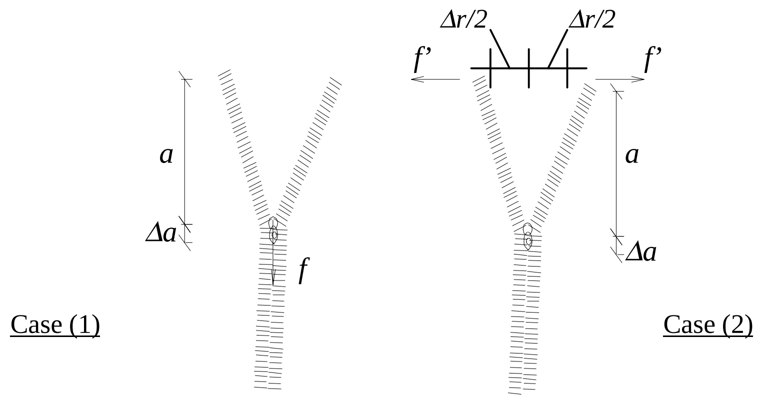


Figure 4 – Zipper representing crack opening.

It is appropriate now to consider that to allow the opening an applied force *f* is necessary to pull ligaments apart. Furthermore, it should be considered the fact that the energy due to *f* is related to the strain energy release rate G. Another possibility would be apply two identical forces (and opposite direction) *f'* and with the same intention to open the zipper. The first procedure is associated here with the crack extension while the second one is given with respect to crack widening. When the crack is extended by Δa (small) for the second case, and the crack is widened of Δr*, one could use a comparison*. Thus, if $\Delta r = 2\Delta a$ and *lim* $\Delta a, \Delta r \to 0$ the energy release due to crack widening could be related to the energy release due to crack extension as follows:

$$\frac{\partial U}{\partial r} = \frac{1}{2}\frac{\partial U}{\partial a} \tag{13}$$

where U denotes the strain energy of the opened zipper. In order to compute the stress intensity factor K, one can apply the Irwin's relation:

$$G = \frac{K^2}{E} \tag{14}$$

From Eqs. (2), (13) and (14) we obtain:

$$K^2 = 2E\frac{\partial U}{\partial r} \tag{15}$$

For a beam subjected to a bending moment M the corresponding change in energy is given by:

$$\frac{\partial U}{\partial r} = M^2[D] \tag{16}$$

where Δr represents the widening of the crack and D is the beam stiffness reduction:

$$D = \frac{1}{EI_f} - \frac{1}{EI_o} \tag{17}$$

2.3 Cross section – I

For I-shaped section case, damage effects, the damaged effect into the bar should be verified following geometric changes during the crack propagation. Thus, one divides a generic I-section as follows:

Case 1 - Crack located in the bottom girder flanges (Fig. 5):

$\overline{y_1} = \frac{1}{A_1}\sum A_i y_i$, where:

$A_1 = bf_1.tf_1 + hw.tw + (tf_2 - a).bf_2$

$\sum A_i y_i = bf_1.tf_1\left(\bar{s} + \frac{tf_1}{2}\right) + hw.tw\left(\bar{s} - \frac{hw}{2}\right)$

$\bar{s} = tf_2 - a + hw$

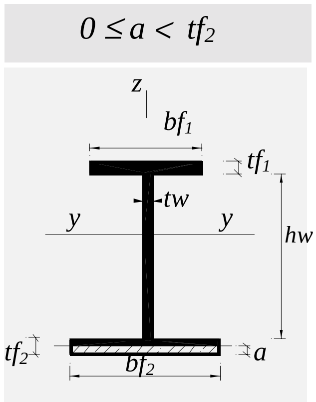


Figure 5 - Crack in the Bottom Flange.

The fractured section moment of inertia is given by:

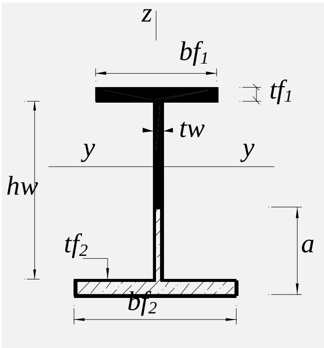
$$I_{y_1} = \frac{1}{12}bf_1.tf_1^3 + bf_1.tf_1.\left(\frac{tf_1}{2} + \bar{s} - \overline{y}_1\right)^2 + \frac{tw}{12}\bar{s}^{3} + tw.\bar{s}.\left(\frac{\bar{s}}{2} - \overline{y}_1\right)^2 \tag{18}$$

Therefore, the damaged section stiffness can evaluate as:

$$k_1 = \frac{EI_o}{\Gamma_1(a(x))} \tag{19}$$

Case 2 - Crack located in the web (Fig. 6):

$tf_2 \leq a < tf_2 + hw$



$$A_1 = bf_1.tf_1 + tw.\bar{s}$$

$$\bar{y}_2 = \frac{[bf_1.tf_1.(\bar{s} + tf_1/2) + \bar{s}^2.tw/2]}{(bf_1.tf_1 + \bar{s}.tw)}$$

$$I_{y_1} = \frac{1}{12} bf_1.tf_1^3 + bf_1.tf_1.\left(\frac{tf_1}{2} + \bar{s} - \bar{y}_2\right)^2 + \frac{tw}{12}\bar{s}^3 + tw.\bar{s}.\left(\frac{\bar{s}}{2} - \bar{y}_2\right)^2 \tag{20}$$

Figure 6- Crack in the Web.

And the damaged section stiffness is given by:

$$k_2 = \frac{EI_o}{\Gamma_2(a(x))} \tag{21}$$

If the worked conjugate forces N - M are considered then ηN must be increased. Thus,

<u>for case 1</u>: $0 \leq a \leq tf_2$

$$\eta_{N1}(a,0) = \frac{a.bf_2}{bf_1.tf_1 + hw.tw + (tf_2 - a)bf_2} \tag{22}$$

<u>for case 2</u>: $tf_2 \leq a \leq tf_2 + hw$

$$\eta_{N2}(a,0) = \frac{tf_2.bf_2 + tw(a - tf_2)}{bf_1.tf_1 + tw.\bar{s}_2} \tag{23}$$

3. ELASTIC PLASTIC STIFFNESS

The derivation of the elastic-plastic stiffness of the cracked element follows the classical plasticity theory, assuming a simple associated flow and isotropic hardening rule. The generalized strain rate is then decomposed into elastic and fully plastic parts as (Rice, 1972):

$$d\mathbf{q} = d\mathbf{q_e} + d\mathbf{q_p} \tag{24}$$

From Eq. (24) one can express the actual generalized stress tensor increment given in terms of efforts (dQ = dN, dM). The elastic strain rate will be given by:

$$\mathbf{dQ} = \Psi(d\mathbf{q_e} - d\mathbf{q_p}) \tag{25}$$

where Ψ represents the elastic compliances and

$$d\mathbf{q_p} = d\Lambda\{\partial f/\partial \mathbf{Q}\} \tag{26}$$

is the plastic strain rate defined according to an adopted yield surface $f(\mathbf{Q}, \alpha_y)$, being $d\Lambda$ the plastic multiplier.

Following the standard plasticity theory, using therefore the flow and hardening rules, one can find the incremental expression of the plastic multiplier (satisfying also the Kuhn-Tucker conditions and consistency condition):

$$d\Lambda = \frac{\left(\frac{\partial f^T}{\partial Q}.\psi\right)^T.dq}{\left(\frac{\partial f^T}{\partial Q}.\psi.\frac{\partial f}{\partial Q} - \frac{\partial f}{\partial \alpha_y}.\frac{E_p}{\sigma_y \chi \eta}.Q.\frac{\partial f^T}{\partial Q}\right)} \tag{27}$$

From Eq. (27) together with Eq. (25) the final expression of the actual effort increment can be found:

$$\mathrm{d}\mathbf{Q} = \psi_{ep}\mathrm{d}\mathbf{q} \tag{28}$$

where ψ_{ep} is the modified values of the elastic compliances.
For the case of I-section beam (see Fig. 7), it is assumed elastic-perfectly plastic behaviour and plane cross section after loading.

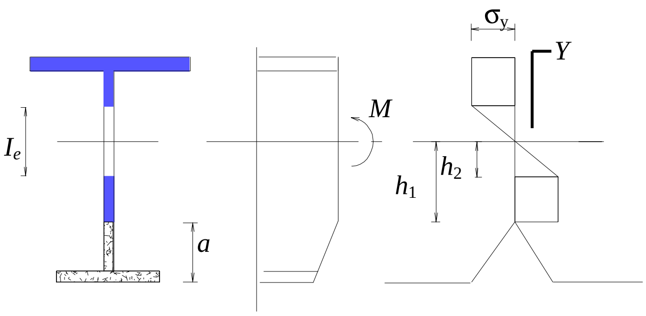


Figure 7 - Cracked Cross Section (elastic-perfectly plastic behavior).

The bending moment is now evaluated as:

$$M = \int \sigma Y dA = \int_{h_1}^{h_2} E\phi Y^2 dA + 2\int_{h_1}^{h_2} E\phi h_1 Y dA = EI_e\phi + 2Eh_1S_1\phi \tag{29}$$

$$\Rightarrow \boxed{M = (k_e + k_p)\phi} \tag{30}$$

It is then considered an approximation assuming an additive decomposition of the elastic and plastic stiffness, i.e. $k_e + k_p$ is the total stiffness of the considered section.

The computer code **RAST** (*Rissausbreitung in Stahlträger*), was developed to model the cracked beam element and to compute its stiffness K (see Fig. 8 and 9 for a flexural stiffness case where $\xi(a) = 0.2\, a$).

4. DAMAGE LOCALIZATION AND SEVERITY PREDICTION

The crack position and evaluation can be obtained through the closed-form solution of the equation of motion. The natural frequency of the cracked beam will be modified resulting to: $\omega_f = \omega_n - \Delta\omega$, where ω_n is the natural frequency of the uncracked beam, and $\kappa = [\omega_f^2 (A\rho / EI)]^{1/4}$. Taking into account the appropriate boundary conditions and the displacements equations resulted from the equation of motion, one can find the characteristic equation of the model, used to obtain the natural frequencies changes:

$$\Delta\omega_n = 2\omega_n . \Omega_n(\beta) . K^{-1} \tag{31}$$

where $\Omega_n(\beta)$ is a function that depends only on the crack position. Now, the inverse technique can be applied to recover the equivalent dimensionless stiffness K from the fracture mechanic theory. From Eq. (31), one obtains:

$$\left(\frac{\Delta\omega_1}{\omega_1}\right)\left(\frac{\Delta\omega_2}{\omega_2}\right)^{-1} = \frac{\Omega_1(\beta)}{\Omega_2(\beta)} \tag{32}$$

The Equation (32) is applied to find the crack localisation. The code **EIGFRISS** (*Eigenfrequenz für Rissmodelle*) has been implemented to identify the crack location and size (Fig. 10). Results were compared with numerical solutions computed by using the commercial code ANSYS, adopting to represent the 3D structure (Deus, 1998).

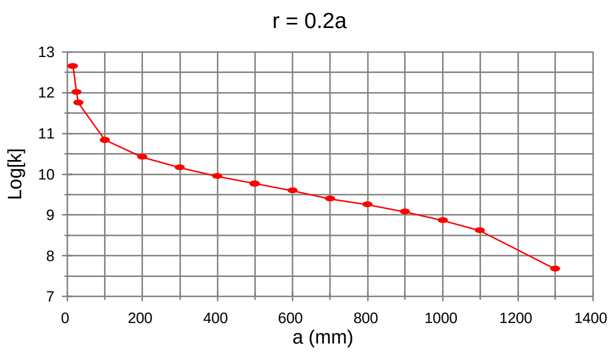


Figure 8 - Crack length(a) x flexural stiffness[K] obtained with RAST- $\xi(a) = r = 0.2a$.

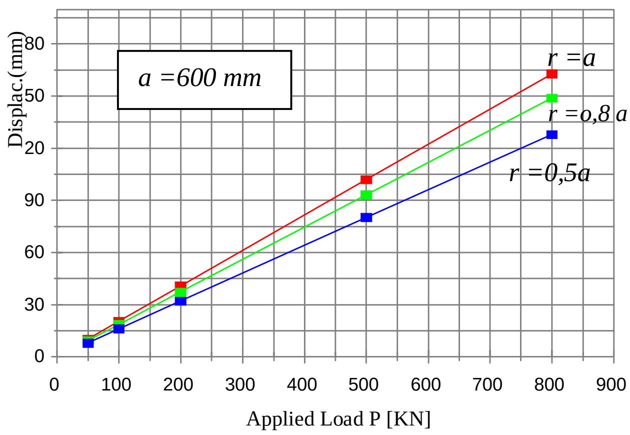


Figure 9 – Crack length linked to the stiffness [K].

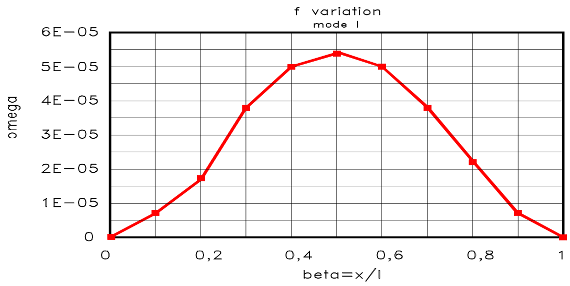


Figure 10- Function $\Omega(\beta)$ versus position β=x/L for simply supported beam.

Table 1. Displacements at beam center for a rectangular cross section.

H/b	L/h	A(mm^2)	I(mm^4)	σ_y(kN/mm^2)		E(kN/mm^2)	P(kN)
2.00	20.00	20,000.00	0.667x10^8	0.25		205.00	1.00
ANSYS				**RAST**			
a/H	0.0	0.333	0.666	0.0		0.333	0.666
Δ	0.025	0.027	0.039	0.025		0.027	0.042
				k*	.182x10^{23}	.120x10^9	.107x10^8

5. CONCLUDING REMARKS

A FEM – *cracked beam* is developed aiming to simulate and analyze the case of *steel bridge beam* with an open crack (Mode I). In this method the crack is simulated by an equivalent rotational spring. The stiffness of a degrading spring ideally placed in the correspondence of the damaging section should be evaluated for any cross section. It has been also derived a simple procedure to find cracks during bridges inspections; Hence, the natural frequency variation were used to estimate crack position and size. The numerical algorithms RAST and EIGFRISS were written to quantify the accuracy of the theoretical damaged models. Regarding advantages and efficiency the proposed model is really practical, flexible to be applied to others structural problems and require simple numerical tools to be used effectively. The results obtained show a remarkable accuracy when compared with other numerical solutions.

6. REFERENCES

Brinckerhoff, P. ,1993, Bridge inspection and rehabilitation, John Wiley & sons, inc.

Deus, E.P. ,1997, Análise do processo de fraturamento em vigas de pontes de aço sob efeito de fadiga, Tese de doutorado, Escola de Engenharia de São Carlos, Brazil.

Deus, E.P. , Venturini, W.S and Peil, U. ,1998, A Theoretical model to identify fractures due to fatigue in steel bridges using inverse technique, Fourth World Congress on Computational Mechanics, Buenos Aires, Argentina.

Deus, E.P. , Venturini, W.S and Peil, U. ,1999, Modeling technique used to identify and evaluate cracks in steel beam bridges under fatigue effects, Sixth Pan-American congress of applied mechanics – PACAM, Rio de Janeiro, Brazil, vol. 6, 409-412.

Rice, J.R., 1972, The line spring model for surface flaws. In the surface crack: physical problems and computational solutions, J.L. Swedlow, ed., ASME, NY. 171-185.

Ostachowicz, W.M. and Krawczuk, M. ,1991, Analysis of the effect of cracks on the natural frequencies of a cantilever beam, Journal of sound and vibration, 150, 191-201.

Paris, P.C. and Erdogan, F. ,1960. A critical analysis of crack propagation laws, Journal of Basic Engineering, 85, 528-534.